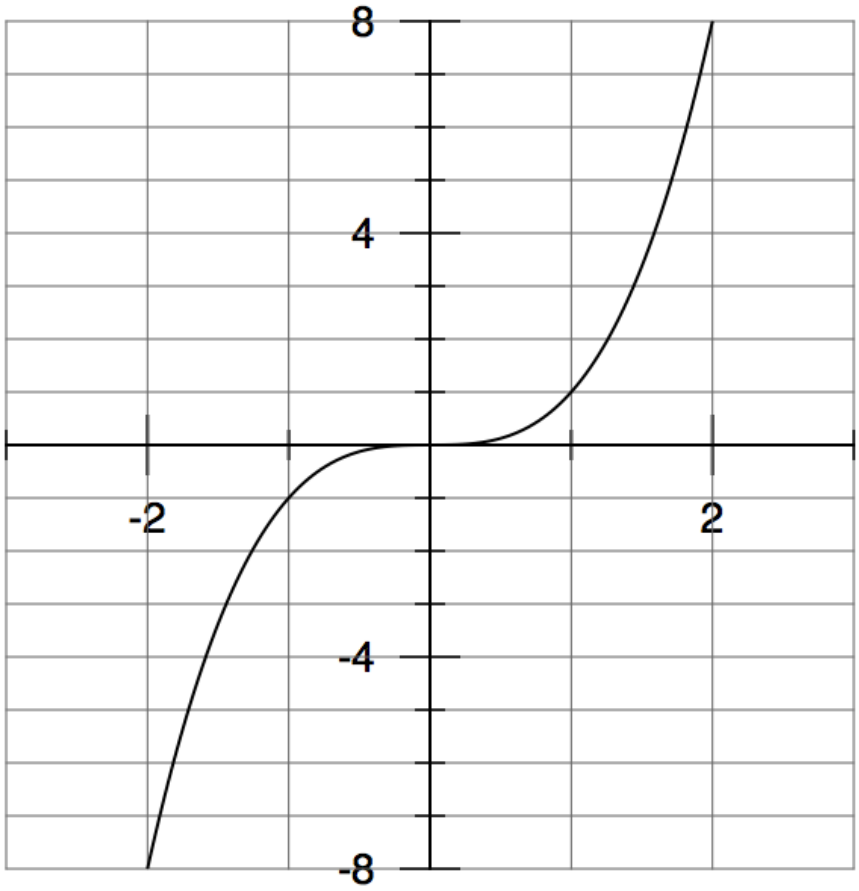
Combining Transformations

These notes are intended as a summary of section 3.4 (p. 219 – 225) in your workbook. You should also read the section for more complete explanations and additional examples.

The graph of $y = x^3$ is shown below.



On the same axes, draw each of the following:

1. The image graph of $y = x^3$ after a translation of 2 units down, followed by a vertical stretch by a factor of 3.

2. The image graph of $y = x^3$ after a vertical stretch by a factor of 3 followed by a translation of 2 units down.

Notice that the 2 resulting graphs are not the same. This illustrates a very important concept:

When two or more transformations are applied to a graph, the order in which the transformations are applied matters!!

How to Combine Transformations

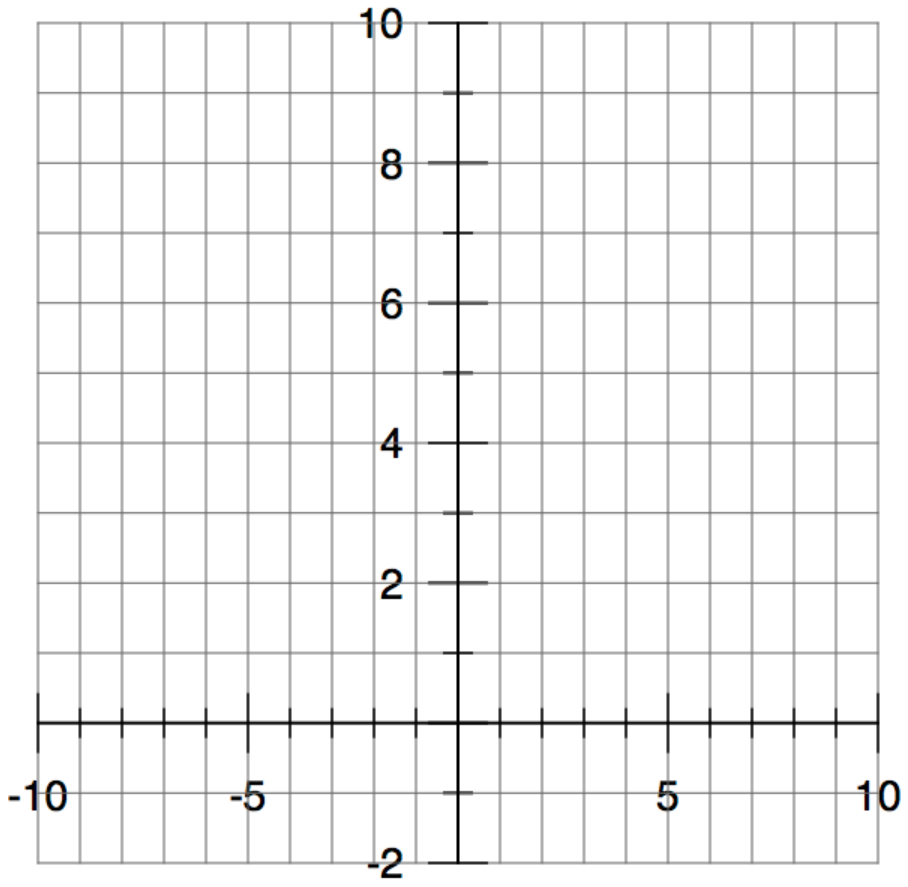
When drawing a graph from an equation, to ensure the correct image is sketched, always perform the transformations in the following order:

1. Stretches or Compressions
2. Reflections
3. Translations

Note: This order assumes that the equation is written in the form $y = a \cdot f\left(b\left(x-h\right)\right)+k$. If it is not, you must first rewrite the equation in the correct form.

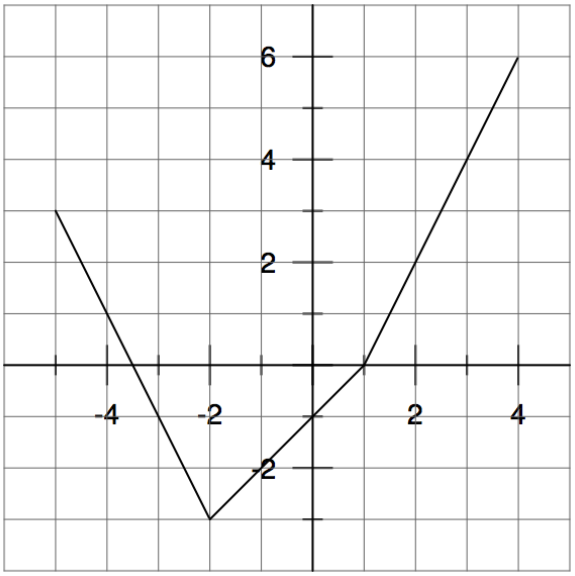
Note: In general, every point $\left(x,y\right)$ on the original graph will become a point $\left(\frac{x}{b}+h, ay+k\right)$ on the image graph.

To see how this is done, let's draw the graph of $y = \frac{1}{4}\left|2\left(x+3\right)\right|+4$.



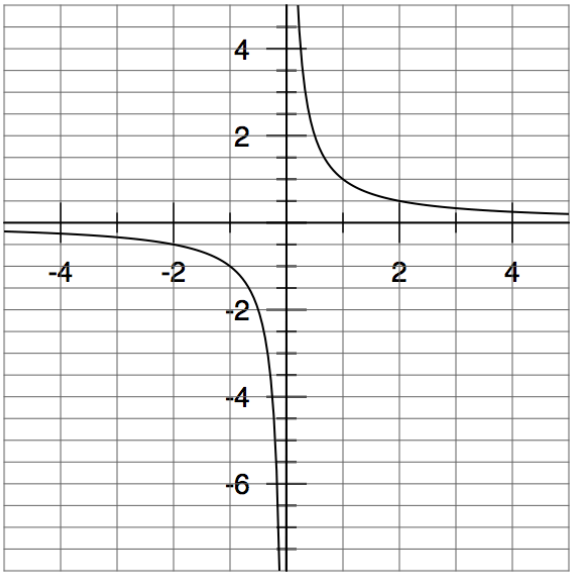
Example 1 (sidebar p. 221)

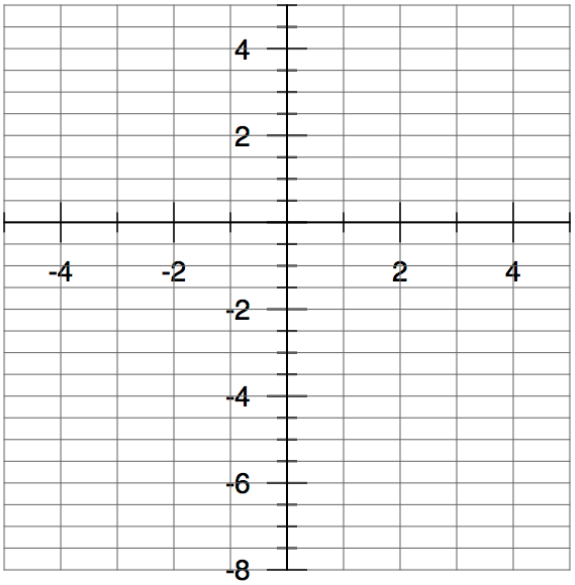
Here is the graph of $y = g(x)$. Sketch and label its image after a vertical compression by a factor of $\frac{1}{3}$, then a translation of 2 units up.



Example 2 (sidebar p. 223)

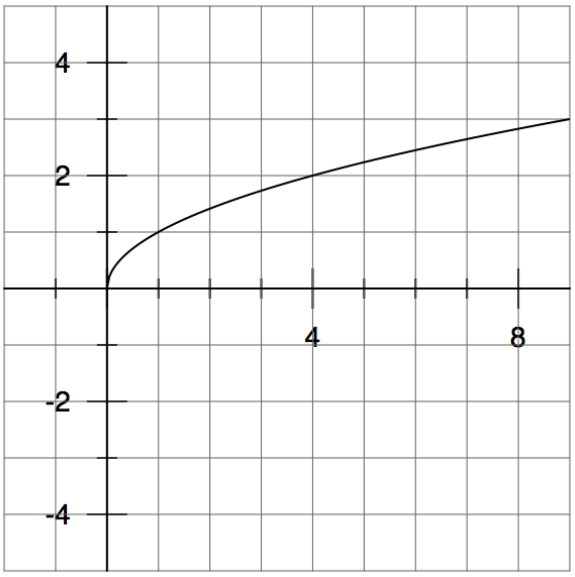
Here is the graph of $y = f(x)$. Sketch the graph of $y = f\left(\frac{1}{2}(x+1)\right) - 4$. State the domain and range of each function.





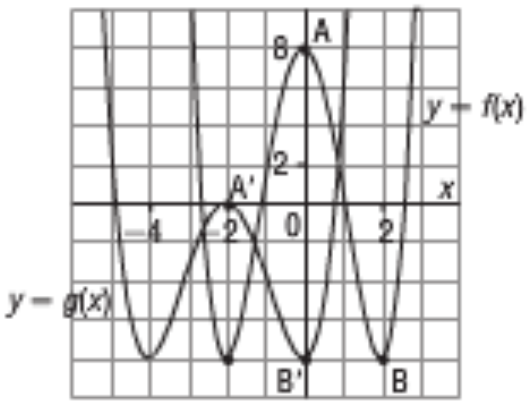
Example 3 (sidebar p. 224)

Use the graph of $y=\sqrt{x}$ to graph $y-2=-\sqrt{3x+3}$. What are the domain and range of $y-2=-\sqrt{3x+3}$?



Example 4 (sidebar p. 225)

The graph of $y=g(x)$ is the image of the graph of $y=f(x)$ after a combination of transformations. Corresponding points are labelled. Write then verify an equation for the image graph in terms of the function f.



Homework: #3 – 11 in the exercises (p. 226 – 233). Answers on p. 233.